Taming.io - T.A.M.E.R Exam

LapaMauve

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Abstract

The questions on the T.A.M.E.R exam from taming.io to obtain badges and achievements can sometimes be difficult to understand. Here is a correction for the most difficult MCQ exercises.

1 Exercise: Increasing the chance of taming

"How many Stones do you need to spend on average so that the probability of Vulture taming is exactly 77% with a Slingshot while wearing the Lucky Hat?"

This is arguably the hardest question in the *T.A.M.E.R exam*, as there are two aspects to this question: **gathering information and problem-solving**.

Firstly, let's try to understand the context. We want to know how many stones, on average, one would need to increase the taming probability of a vulture to **77%**.

Indeed, when you shoot certain Tamons (like the vulture) with a slingshot, you increase the taming success probability by **1%**. However, and this is important, if you miss your shot, the bonus probability resets to **0%**.

Furthermore, there are several ways to artificially increase this probability, such as the *Tamer's Cap* and potions like the *Elixir of Influence* or *the Brew of Unbreakable Friendship*. But in this question, it is explicitly mentioned that we are not using the *Tamer's Cap* and, by implication, that we are not using any potion, as there is no reference to potions. When the exam doesn't mention specific potions, it is understood that no potions are in play.

Thus, we first need to find out how many times we must successively and flawlessly hit the vulture with a slingshot to increase its taming bonus so that its base taming chance plus the bonus equals **77%**.

To find out the base taming chance, the easiest way is to take a slingshot, go to the desert, and shoot a baby vulture.

You will get **11%** when you successfully hit it. When you shoot, you increase the base taming by **1%**, so the calculation is:

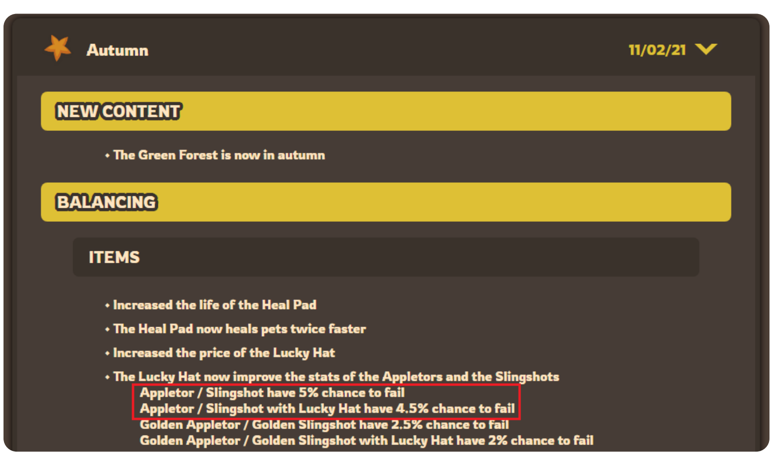
$$11\% - 1\% = 10\%$$

Therefore, the baby vulture's base taming chance is indeed **10%**. Thus, to find the number of successful consecutive shots with a slingshot, one must calculate:

$$77\% - 10\% = 67\%$$

Remark 1: *More difficult variants of this exercise exist and require knowledge of the tame base for legendary Tamons which are not necessarily available in the game. You're out of luck! But you can find this figure on the numerous youtube videos where you can see other tamers trying to tame these animals.*

Next, and this is crucial, you must know your chances of a successful shot with a slingshot to increase a Tamon's taming bonus probability while wearing a Lucky Hat. The Lucky Hat, among other things, decreases the probability of missing a shot with a Slingshot or Appletor. To gather this information, one needed to check the taming changelog, specifically in the description of the *Autumn* update on *October 2, 2021*.

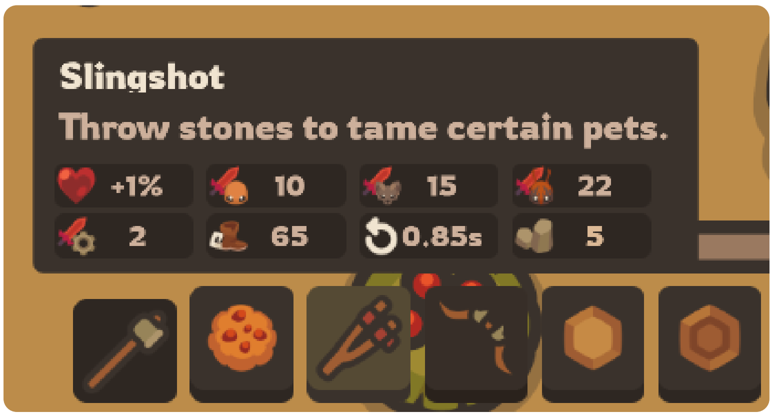


From this source, we obtain the desired information: the failure probability is **4.5%**, meaning the success probability is:

$$100\% - 4.5\% = 95.5\%$$

Remark 2: *here exists a variant of this question without the Lucky Hat, and the same changelog page informs you that the failure rate under those conditions is **5%**, or a **95%** success rate.*

Finally, in this question, we need to know the stone cost of a shot with the slingshot, which is 5 stones per shot. To find this, one simply hovers the mouse (or finger) over the slingshot in the game. Note that there's also a cost of 5 apples for an Appletor, which could be used in a variant of this question.



Thus, we have gathered all the necessary information for solving this exercise:

- **67** successful consecutive shots with the Slingshot are required.
- The probability of a successful shot is **95.5%**.
- The cost of one shot is **5 stones**.

Problem solving: Now, let's solve the given problem. For this, we need to understand what is meant by *"on average."*. Let's make an analogy with a fair coin with 2 a side called *Heads* and a side called *Tails*.

If you flip a balanced coin (i.e., a coin where the probability of getting heads or tails is the same and equals 0.5), the probability of getting *heads* in a single flip is 0.5.

The average number of tries to get *heads* is indeed 1 divided by the probability of getting *heads*, or:

$$\text{Average number of tries} = \frac{1}{0.5} = 2$$

Thus, on average, you would need 2 tries to get *heads* by flipping a balanced coin.

In other words, if you flip a coin a large number of times, say 1000 times, and you count the number of times you got *heads*, you would probably get around 500. If you want to know how many flips it took on average to get *heads*, you'll compute:

$$\frac{1000}{500} = 2$$

On average, you indeed need 2 flips to get *heads*.

Now, let's redo this reasoning to obtain the average number of coin flips to get two consecutive *heads*. Let's call this value H^2.

You start by flipping the coin once; you have a 0.5 chance of getting *Tails* and a 0.5 chance of getting *Heads*.

- If you get Tails with a probability of 0.5, you must start the H^2 experiment from the beginning, which translates to $0.5 \times (H^2 + 1)$. We add 1 because we have flipped a coin once.
- If you get *Heads*, then you repeat the flip, hoping to get *Heads* again.

At this point in the formula, we can write:

$$H^2 = 0.5 \times (1 + H^2) + 0.5 \times (\ldots)$$

If we continue and redo the flip and get Tails with a 50% chance, we must restart the entire process, which is $0.5 \times (2 + H^2)$. We add 2 because we've flipped two coins, and if we get *Heads*, we have succeeded with a 0.5×2. Again, we have 2 here because it's the second coin we've flipped. Completing the above formula, we get:

$$\begin{aligned}
H^2 &= 0.5 \times (1 + H^2) + 0.5 \times \big(0.5 \times (2 + H^2) + 0.5 \times 2\big) \\
&= 0.5 + 0.5H^2 + 0.5 + 0.25H^2 + 0.5 \\
&\Rightarrow H^2 = 0.75H^2 + 1.5 \\
&\Rightarrow 0.25H^2 = 1.5 \\
&\Rightarrow H^2 = \frac{1.5}{0.25} \\
&\Rightarrow H^2 = 6
\end{aligned}$$

Let's see what it gives with H^3, that is, the average number of throws to get three consecutive Heads. We perform the same reasoning and obtain the formula:

$$H^3 = 0.5 \times (1 + H^3) + 0.5 \times \big(0.5 \times (2 + H^3) + 0.5 \times \big(0.5 \times (3 + H^3) + 0.5 \times 3\big)\big)$$

Then, we get

$$H^3 = 14$$

The most important thing is to see the recursion that is starting to appear. As an exercise, you can try to find H^4 before continuing.

$$\begin{aligned}
&H^4 = 0.5 \times (1 + H^4) + 0.5 \times \big(0.5 \times (2 + H^4) + 0.5 \times \big(0.5 \times (3 + H^4) + 0.5 \times \big(0.5 \times (4 + H^4) + 0.5 \times 4\big)\big)\big) \\
&H^4 = 30
\end{aligned}$$

Now that you understand how it works for a balanced coin, we can generalize the concept and do it for something other than a balanced coin, such as the success rate of a slingshot shot! But as this exercise has many variants, we will use abstract variables instead.

Here, we use:

$p =$ chance of a successful slingshot shot with the Lucky Hat
$q =$ chance of a failed slingshot shot with the Lucky Hat
S^k the number of k successful consecutive slingshot shots.

So as not to confuse you, here are the real values of p and q:

$$p = 95.5\% = 0.955$$

$$q = 1 - p = 1 - 0.955 = 0.045$$

For one successful shot, you must use this formula to get the average number of shots (it's the same reasoning as for a coin toss):

$$S^1 = \tfrac{1}{p} \approx 1.05$$

For two consecutive successful slingshot shots, we have the same reasoning as for two consecutive heads on a coin toss, but this time using p and q.

$$\begin{aligned}
S^2 &= q \times (1 + S^2) + p \times \big(q \times (2 + S^2) + p \times 2\big) \\
S^2 &= S^2 \times (q + pq) + (q + 2pq + 2p^2) \\
S^2 &= \frac{q + 2pq + 2p^2}{1 - (q + pq)}
\end{aligned}$$

We apply the same reasoning for three consecutive shots:

$$\begin{aligned}
S^3 &= q \times (1 + S^3) + p \times \big(q \times (2 + S^3) + p \times \big(q \times (3 + S^3) + p \times 3\big)\big) \\
S^3 &= \frac{q + 2pq + 3qp^2 + 3p^3}{1 - (q + pq + qp^2)}
\end{aligned}$$

We quickly understand the recurrence and finally obtain the formula:

$$\boxed{S^k = \frac{kp^k + q \times \sum_{i=1}^{k}(i \times p^{i-1})}{1 - q \times \sum_{i=0}^{k-1} p^i}}$$

We can trivially demonstrate its validity with structural induction on the set of natural numbers $\mathbb{N}$. We have already initialized it, only the inheritance remains. *I will leave this triviality to our reader.*

Now we found the formula, let's continue the simplificaiton, in the denominator, we recognize a simple geometric series.

$$S^k = \frac{kp^k + q \times \sum_{i=1}^{k}(i \times p^{i-1})}{1 - q \times \frac{1-p^k}{1-p}}$$

In the numerator, we recognize the derivative of the following geometric series:

$$\frac{d}{dp}\left(\sum_{i=0}^{k} p^{i+1}\right) = \left(\sum_{i=1}^{k} i \times p^{i-1}\right) + (k+1) \times p^k$$

We then introduce the following component in place of the numerator:

$$\frac{d}{dp}\left(p \times \frac{1 - p^{k+1}}{1 - p}\right) - (k+1) \times p^k$$

Finally, we obtain the following formula simplified from all its sums:

$$S^k = \frac{kp^k + q \times \left(\frac{1-p^{k+1}}{1-p} + p \times \frac{-p^k(k+1)(1-p) + (1-p^{k+1})}{(1-p)^2} - (k+1) \times p^k\right)}{1 - q \times \frac{1-p^k}{1-p}}$$

We use that $q = 1 - p$ and $p = 1 - q$, so we replace these terms :

$$
\begin{aligned}
S^k &= \frac{kp^k + q \times \left(\frac{1-p^{k+1}}{q} + p \times \frac{-p^k(k+1)q+(1-p^{k+1})}{q^2} - (k+1) \times p^k\right)}{1 - q \times \frac{1-p^k}{q}} \\
&= \frac{kp^k + 1 - p^{k+1} - p^{k+1}(k+1) + \frac{p-p^{k+2}}{q} - (k+1) \times p^k \times q}{p^k} \\
&= k - p - p \times (k+1) - \frac{p^2}{q} - (k+1) \times q + \frac{1 + \frac{p}{q}}{p^k} \\
&= -1 - p - \frac{p^2}{1-p} + \frac{1 + \frac{p}{1-p}}{p^k} \\
&= \frac{(p+1) \times (p-1) - p^2 + (1-p) \times p^{-k} + p \times p^{-k}}{1-p} \\
&= \frac{p^{-k} - 1}{1-p}
\end{aligned}
$$

One last simplification and we get the beautiful and straightforward formula:

$$
\boxed{S^k = \frac{p^{-k} - 1}{q}}
$$

Let's now substitute the value of k with **67**, the value of p with **0.955**, and the value of q with **0.045** into the previous formula:

$$
S^{67} = \frac{(0.955)^{67} - 1}{0.045} \approx 463.7
$$

We round up to the next integer because we want to get a real number of slingshot shots.

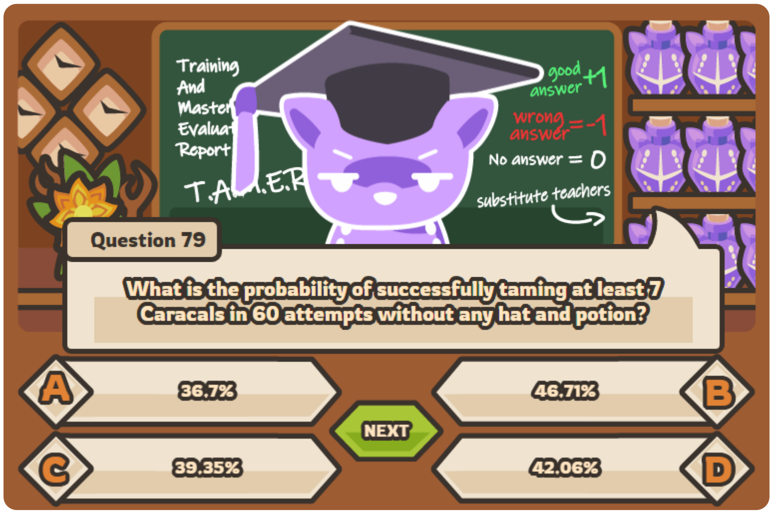
$$
\lceil S^{67} \rceil = 464
$$

And we finally multiply the result by the number of stones needed to take a slingshot shot.

$$
\boxed{\lceil S^{67} \rceil \times 5 \text{ Stones} = 464 \times 5 \text{ Stones} = 2320 \text{ Stones}}
$$

So the correct answer here was **B**

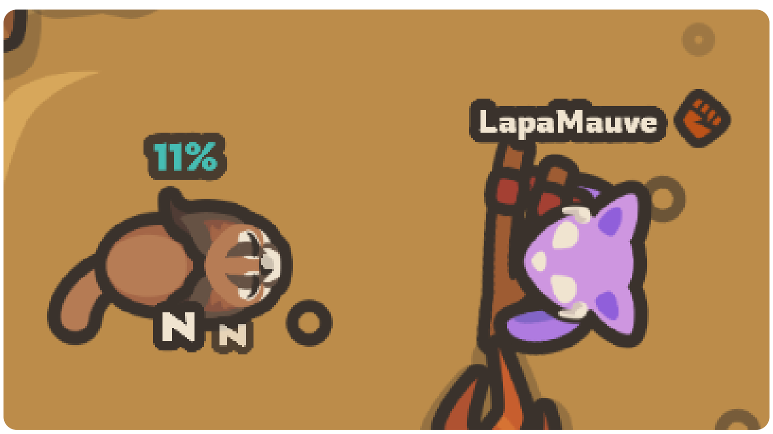
2 Exercise: Taming success rate



"What is the probability of successfully taming at least 7 Caracals in 60 attempts without any hat and potion?"

This question is less challenging than the previous one, provided you are familiar with the binomial distribution.

As we saw earlier, you can deduce that the base taming rate of the Caracal is **10%** again. And once again, the question explicitly states that no potions or hats are involved.



In this question, we repeat the same random experiment **60 times**. The repetitions are independent, and each experiment *(taming attempts of caracals)* can result in two opposing outcomes: success, with a certain probability, or failure.

The random variable X, representing the number of successes, then follows a binomial distribution over the set $\{0, ..., 60\}$, with parameter $p = 10\% = 0.1$. This means that:

$$\mathbb{P}(X = k) = \binom{60}{k} \times 0.1^k \times (1 - 0.1)^{60-k}$$

Remark 1: The symbol $\binom{60}{k}$ represents the binomial coefficient, which gives the number of ways to choose k successes out of 60 trials.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The subtlety of this question is that we want to know the probability of successfully taming **at least 7 caracals**. This probability thus encompasses the chances to tame 8, 9, 10, all the way up to 60 caracals!

So the result becomes:

$$\boxed{\mathbb{P}(X \geq 7) = \sum_{k=7}^{60} \binom{60}{k} \times 0.1^k \times (1-0.1)^{60-k}}$$

An approximation of the previous formula gives us:

$$\boxed{\mathbb{P}(X \geq 7) \approx 39.35}$$

So the correct answer here was **C**